

Schutz

6.7. (a) Give the definition of the determinant of a matrix A in terms of cofactors of the elements.

For matrix A , pick any row, or column, compute

$$\sum a_{ij} (-1)^{i+j} C_{ij}$$
 where a_{ij} is the (i,j) element, C_{ij} is the ij cofactor, along this row, the result is the determinant of A .

Or, let $C(g_{\alpha\beta})$ denote the cofactor matrix of $g_{\alpha\beta}$,

$$\text{we have } \frac{C(g_{\alpha\beta})^T}{\det(g_{\alpha\beta})} = g^{\alpha\beta}$$

$$\Rightarrow C(g_{\alpha\beta})^T g_{\alpha\beta} = \det(g_{\alpha\beta})$$